Parameter estimation using smoother techniques

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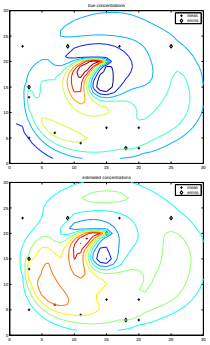
Twin experiment

- 2D advection-diffusion equation discretized on 30×30 grid
- Chemistry scheme describing the conversion of SO_2 to SO_4

Some locations of constant emissions of SO_2
Some SO_4 observation locations



Model

The stochastic emissions and observations are modelled using white noises.

$$\begin{bmatrix} c(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} \mathcal{A}(k) & \mathcal{B}(k) \\ 0 & I \end{bmatrix} \begin{bmatrix} c(k) \\ e(k) \end{bmatrix} + \begin{bmatrix} 0 \\ w(k) \end{bmatrix}$$

$$x(k+1) = \mathcal{M}(k)x(k) + W(k)$$

$$y^0(k) = H(k)x(k) + v(k)$$

Ensemble Smoother Approaches

Ensemble fixed-lag smoother

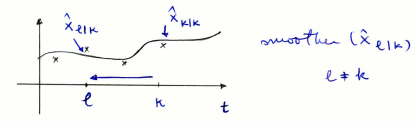
We redefine the state vector as:

$$Y(k) = \begin{bmatrix} x(k) \\ x(k-1) \\ \vdots \\ x(k-N) \end{bmatrix} \Rightarrow Y(k+1) = \begin{bmatrix} \mathcal{M}(k) & 0 & \dots & 0 \\ I & 0 & \dots & 0 \\ 0 & I & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & I \end{bmatrix} Y(k) \dots$$

and the observation operator:

$$y^0(k) = \begin{bmatrix} \mathbf{H}(k) & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \\ \vdots \\ x(k-N) \end{bmatrix} + v(k)$$

EnKS and FIFO



The filter analysis at time l in terms of the ensemble covariances matrices P_{ens}:

$$A^a = A^f + P_{ens}H^T(HP_{ens}H^T + R_{ens})^{-1}(D - HA)$$

$$A^a = A^f + A'A'^TH^T(HA'A'^TH^T + R_{ens})^{-1}(D - HA)$$

The EnKS analysis for a prior time l and $k > l$:

$$A^a(l) = A^f(l) + A'(l)A'^T(k)H^T(HA'A'^TH^T + R_{ens})^{-1}(D - HA)$$

Some results

It has two parts:

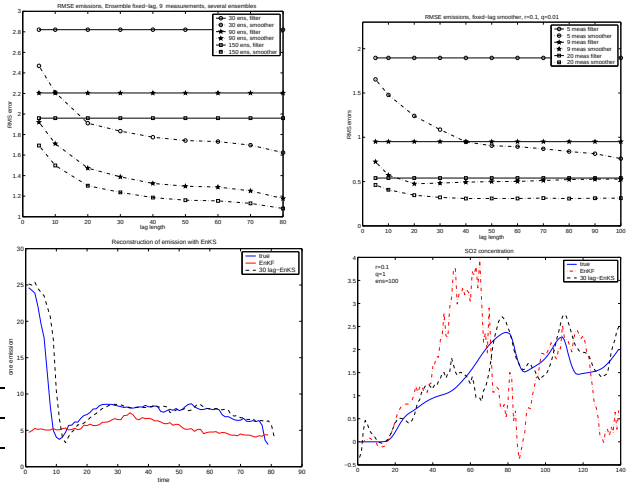
- estimation of SO_4 and SO_2 concentrations
- estimation of SO_2 emissions

Two algorithms are used:

- Ensemble fixed-lag smoother (EnKS)
- FIFO (Ravela and McLaughlin)

		SO_4	SO_2	emiss
$r = 1$ and $q = 0.1$	filter	0.16	0.66	0.58
	30 ENKS	0.08	0.32	0.31
$r = 1$ and $q = 10$	filter	0.38	1.41	0.96
	30 ENKS	0.35	0.23	0.84

100 ens	filter	10fixed-lag	30fixed-lag	60fixed-lag	90fixed-lag
time(min)	2.89	3.33	5.52	8.51	12.5



Conclusions

Accuracy

- The quality of the smoothed estimates was studied in relation with different sources of uncertainties : model errors, measurement errors.
- Smoother algorithms improve SO_4 and SO_2 concentrations and are able to reconstruct emission of pollutant.

Efficiency

- The time effort for the smoother is larger comparing to the filter and depends on lag length.
- FIFO algorithm based on EnKS is 2-3 time faster than EnKS.